

# Chapter 2 Closure What have I learned?

## Reflection and Synthesis

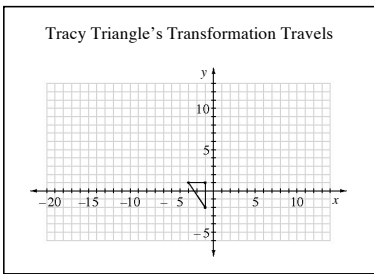
The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.



### ① SUMMARIZING MY UNDERSTANDING

This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will give you directions for exactly how to do this and will provide you with instructions about how to summarize your understanding of transformations and **undoing** transformations. In this activity, you will use a triangle to review transformations.

**Predict and Order:** Predict how each transformation will change or move the shape. Select an order for the four transformations and predict what the new coordinates of the vertices will be after each step.



**Apply Transformations:** Follow the transformation steps you described on the graph. Use color and written descriptions to show how each transformation alters the shape and its position on the coordinate grid. Check that the coordinates you predicted were correct.

<b>Tracy Triangle's Transformative Travels</b> Add 5 to each $x$ -coordinate	<b>Tracy Triangle's Transformative Travels</b> Multiply each $y$ -coordinate by $-1$ .
<b>Tracy Triangle's Transformative Travels</b> Multiply each coordinate by 2	<b>Tracy Triangle's Transformative Travels</b> Add $-8$ and then 3 to each $x$ -coordinate Add 9 and $-4$ to each $y$ -coordinate

*Problem continues on next page. →*

- ① *Problem continued from previous page.*

**Undo Transformations:** To get the triangle back to the original position, **undo** your transformations. You may choose to **undo each** transformation step or to find a new series of steps to return the shape to its original position. Show, using color, symbols, and written descriptions, how each transformation changes the shape's size and position on the coordinate grid.

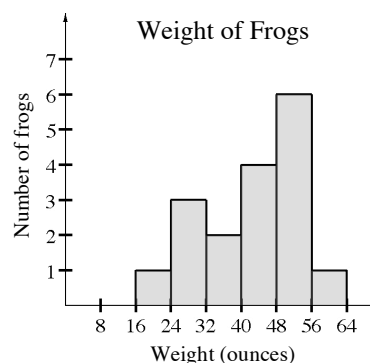
- ② **WHAT HAVE I LEARNED?**

Working the problems in this section will help you to evaluate which types of problems you feel comfortable with and which ones you need more help with.

Solve each problem as completely as you can. The table at the end of this closure section provides answers to these problems. It also tells you where you can find additional help and practice on problems like them.

CL 2-156. The histogram at the right shows the weights of the frogs entered in the Jumping Frog Jubilee.

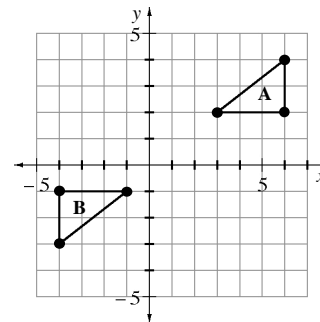
- How many frogs weigh more than two pounds (32 ounces)?
- What is the most common weight for the frogs?
- What is the weight of the largest frog(s)?



CL 2-157. Find the mean, mode, range, and median of the values:

8, 13, 6, -1, 10, 0, -3, 5, 7, 1

CL 2-158. Copy the graph at right on your paper. You will need a second graph for List II. Complete each list of transformation steps you could use to move triangle B back to where it started at position A, and show each transformation on your graph.

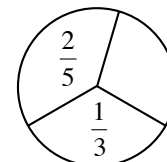


List I	List II
1. Rotate triangle B $180^\circ$ about point $(-1, -1)$	1. Reflect triangle B across the y-axis
2. ?	2. ?
	3. ?

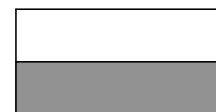
CL 2-159. Neatly graph the points  $(-2, 9)$ ,  $(-3, 7)$  and  $(-5, 10)$  on a four-quadrant graph. Connect them to make a triangle. Then, for each transformation described below:

- Write and simplify an expression to find the new coordinates.
  - Check your answer on your graph.
- a. Slide the triangle right 4 units and down 6 units.
  - b. Reflect the triangle across the y-axis.

CL 2-160. George is playing a game, but the spinner is incomplete. Help him figure out what fraction is missing.



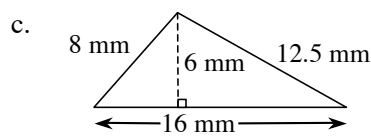
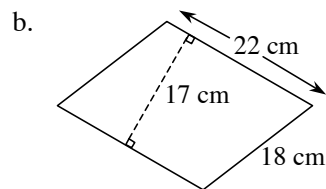
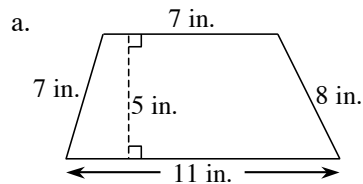
CL 2-161. Multiply  $\frac{2}{3} \cdot \frac{1}{2}$ . You may want to copy and complete the diagram at right to show your work.



CL 2-162. Mary has a bag of colored tiles. There are 8 red tiles, 7 blue tiles, 9 yellow tiles, and 12 green tiles. If she reaches into the bag, what is the probability of picking a:

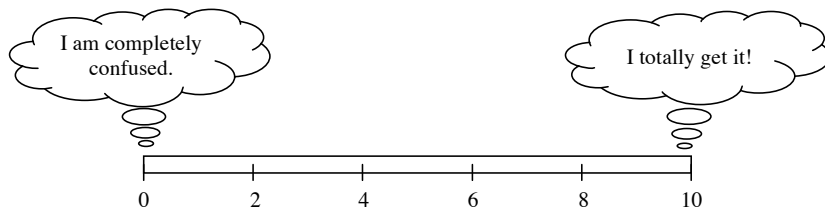
- a. Yellow tile?
- b. Green tile?
- c. Purple tile?

CL 2-163. Copy each shape below on your paper, then find its area and perimeter. Show all your work.



CL 2-164. For each of the problems in this section of closure, do the following:

- a. Draw a bar or number line like the one below that represents 0 to 10.



- b. Color or shade in a portion of the bar that represents your current level of understanding and comfort with completing that problem on your own.
- c. If any of your bars are less than a 5, choose *one* of those problems and do one of the following tasks:
- Write two questions that you would like to ask about that problem.
  - Brainstorm two things that you **DO** know about that type of problem.
- d. If all of your bars are a 5 or above, choose one of those problems and do one of these tasks:
- Write two questions you might ask or hints you might give to a student who was stuck on the problem.
  - Make a new problem that is similar and more challenging than that problem and solve it.

### ③ WHAT TOOLS CAN I USE?

You have several tools and references available to help support your learning – your teacher, your study team, your math book, and your Toolkit, to name only a few. At the end of each chapter you will have an opportunity to review your Toolkit for completeness as well as to revise or update it to better reflect your current understanding of big ideas.

The main elements of your Toolkit should be your Learning Log, Math Notes, and the vocabulary used in this chapter. Math words that are new to this chapter appear in bold in the text. Refer to the lists provided below and follow your teacher's instructions to revise your Toolkit, which will help make it a useful reference for you as you complete this chapter and prepare to begin the next one.



#### **Learning Log Entries**

- Lesson 2.1.2 – Meanings for –
- Lesson 2.1.3 – Subtraction of Integers
- Lesson 2.2.3 – Rigid Transformations
- Lesson 2.3.2 – Rearranging Shapes to Find Area
- Lesson 2.3.4 – Area of Parallelograms and Triangles
- Lesson 2.3.6 – Area of Trapezoids

#### **Math Notes**

- Lesson 2.1.1 – Commutative and Associative Properties of Addition
- Lesson 2.2.1 – Adding and Subtracting Integers
- Lesson 2.2.2 – Graphing Points on an  $xy$ -Coordinate Grid
- Lesson 2.2.3 – Rigid Transformations
- Lesson 2.2.4 – Mixed Numbers and Fractions Greater Than One
- Lesson 2.3.2 – Base and Height of Rectangles
- Lesson 2.3.3 – Parallelogram Vocabulary
- Lesson 2.3.4 – Area of a Parallelogram
- Lesson 2.3.5 – Area of a Triangle
- Lesson 2.3.6 – Order of Operations



### Mathematical Vocabulary

The following is a list of vocabulary found in this chapter. Some of the words have been seen in the previous chapter. The words in bold are the words new to this chapter. Make sure that you are familiar with the terms below and know what they mean. For the words you do not know, refer to the glossary or index. You might also want to add these words to your Toolkit for a way to reference them in the future.

area	<b>associative property</b>	<b>commutative property</b>
<b>coordinate grid</b>	<b>coordinates</b>	<b>expression</b>
<b>integers</b>	<b>lengths</b>	<b>mixed numbers</b>
<b>origin</b>	<b>parallelogram</b>	<b>quadrant</b>
<b>quadrilateral</b>	<b>reflection</b>	<b>rigid transformation</b>
<b>rotation</b>	<b>term</b>	<b>translation</b>
<b>trapezoid</b>	<b>triangle</b>	<b>vertex</b>
<b>x-axis</b>	<b>y-axis</b>	

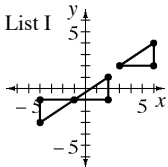
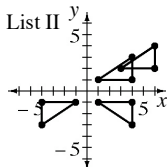
### Process Words

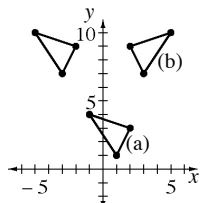
The list of words below are problem solving strategies and processes that you have been involved in throughout the course of this chapter. Make sure you know what it means to do each of the following. If you are not sure, look through your book for problems when you were asked to think in the following ways.

brainstorm	choose a strategy	decompose
describe	explain your reasoning	justify
predict	rearrange	reverse your thinking
test your prediction	visualize	

## Answers and Support for Closure Activity #2

### *What Have I Learned?*

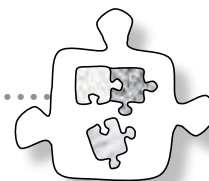
Problem	Solution	Need Help?	More Practice
CL 2-156.	a. 13 frogs b. between 49 and 56 ounces c. between 57 and 64 ounces	Lesson 1.1.4	Problems 1-23, 1-28, 2-88, and 2-111
CL 2-157.	mean: 4.6 median: 5.5 mode: none range: 16	Lessons 1.1.3 and 1.1.4  Math Notes boxes in Lessons 1.1.3 and 1.1.4	Problems CL 1-44, 2-28, 2-79, and 2-141
CL 2-158.	List I: 2. Translate (slide) triangle B right 4 units and up 3 units.  List II: 2. Reflect triangle B across the $x$ -axis. 3. Translate triangle right 2 units and up 1 unit.  List I  List II 	Lessons 2.2.1, 2.2.2, and 2.2.3  Math Notes box in Lesson 2.2.3  Learning Log (problem 2-66)	Problems 2-47, 2-51, 2-52, 2-59 through 2-64, 2-68, 2-85, 2-132, and 2-154
CL 2-159.	a. $x: -2 + 4 = 2$ , $y: 9 - 6 = 3$ ; (2, 3) $x: -3 + 4 = 1$ , $y: 7 - 6 = 1$ ; (1, 1) $x: -5 + 4 = -1$ , $y: 10 - 6 = 4$ ; (-1, 4) b. $y$ -coordinates remain the same. $x: -2 \cdot (-1) = 2$ ; (2, 9) $x: -3 \cdot (-1) = 3$ ; (3, 7) $x: -5 \cdot (-1) = 5$ ; (5, 10)	Lessons 2.2.1, 2.2.2, and 2.2.3  Math Notes box in Lesson 2.2.3  Learning Log (problem 2-66)	Problems 2-47, 2-51, 2-52, 2-55, 2-59 through 2-64, 2-68, 2-77, 2-85, 2-108, 2-132, and 2-154



Problem	Solution	Need Help?	More Practice
CL 2-160.	$\frac{4}{15}$	Lessons 1.2.4 and 1.2.5 Math Notes box in Lesson 1.2.4 Learning Log (problem 1-107)	Problems CL 1-140, 2-9, 2-98, and 2-153
CL 2-161.	$\frac{2}{6}$ or $\frac{1}{3}$	Lesson 2.3.1	Problems 2-93, 2-94, 2-98, and 2-127
CL 2-162.	a. $\frac{9}{36}$ or $\frac{1}{4}$ b. $\frac{12}{36}$ or $\frac{1}{3}$ c. 0: There are no purple tiles; impossible	Lessons 1.2.1 and 1.2.2 Math Notes boxes in Lessons 1.2.1 and 1.2.2	Problems CL 1-139, 2-42, 2-50, 2-71, 2-78, and 2-138
CL 2-163.	a. $A = 45$ sq in., $P = 33$ in. b. $A = 374$ sq cm, $P = 80$ cm c. $A = 48$ sq mm, $P = 36.5$ mm	Lessons 2.3.2 through 2.3.5 Math Notes boxes in Lessons 2.3.2 through 2.3.5 Learning Log (problems 2-106, 2-126, and 2-150)	Problems 2-102, 2-112, 2-115, 2-116, 2-124, 2-128, 2-129, 2-135, 2-137, and 2-151



## Puzzle Investigator Problems



### PI-3. SIERPINSKI TRIANGLES

**Fractals** are geometric structures developed by repeating a process over and over. A famous example of a fractal is the **Sierpinski Triangle**, shown below. To create this design, start with a triangle, as shown in Figure 1. Then find and connect the midpoints of all of the sides of the triangle, subdividing it into four smaller triangles. Shade all but the central triangle as shown in Figure 1.

Then repeat the process by finding and connecting the midpoints of the shaded triangles of Figure 1. Shade all but the center triangles, as shown in Figure 2. If this process is continued infinitely, the result is the Sierpinski Triangle.



Figure 0

Figure 1

Figure 2

Figure 3

- On the PI-3 Resource Page (which can be downloaded at [www.cpm.org/students](http://www.cpm.org/students)), shade in the next figure in the sequence.
- For Figures 1 through 4, write a fraction to represent the amount of the entire triangle that is shaded. As you work, look for patterns. What is happening to the numerator and denominator of the fraction? What is happening to the amount shaded?
- Use your pattern to predict what portion of Figure 5 and Figure 6 in the sequence will be shaded.
- The shaded portion of Figure 2 could be written as  $\left(\frac{3}{4}\right)^2$ . Rewrite the other figures in this form. According to this pattern, what should the value of  $\left(\frac{3}{4}\right)^0$  be? Explain why.
- Is there any figure that will have less than 10% shaded? If so, use your pattern to explain how you know.

PI-4. FAIR SHARES?

Three students baked cookies to share the next day and put them on a plate. However, Latisha woke up in the middle of the night and ate one-third of the cookies and went back to sleep. A little while later, Susan woke up, ate one-third of what was left and fell asleep. Then Hieu woke up, ate one-third of what was left, and went back to sleep. When all three students woke up, they discovered 8 cookies were left.

- a. How many cookies did they bake? Drawing diagrams may help.
- b. What if there had been four students instead? Solve the problem again where four students bake cookies and each of them wakes up separately and eats one-fourth of the cookies that are left. This time, assume that at the end, 81 cookies were on the plate. How many cookies did they bake?
- c. For each situation above, compare the original number of cookies to the final number of cookies left. What is their relationship?